Master 2 internship 2024-2025
A study of the geometric phase of multivariate signals

Keywords Multivariate time series, geometric phase, differential geometry, quantum systems.

Location Grenoble, France.

Supervision

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Application procedure Send a detailed CV and motivation letter to `nicolas.le-bihan@cnrs.fr`, `pierre-olivier.amblard@cnrs.fr`, and `julien.flamant@cnrs.fr`.

Context The concept of geometric phase was discovered by Sir M.V. Berry [1] for quantum systems. It was since extensively studied and has found numerous implications [2]. Many classical systems have also been identified to exhibit geometric phases, amongst them one can find the Foucault pendulum, falling cats or polarized light [3]. Geometric phase can be understood as an excess between the dynamical and relative phase of a system during a time evolution. This physical quantity is purely geometric as it is inherited from the curvature of the underlying parameter space of the Hamiltonian. It arises when a specific path is followed over time in the parameter space. Geometric phase is also gauge-invariant, which makes it desirable for the study of quantum systems [2]. Recently, it was showed in [4] that bivariate signals may exhibit a geometric phase under mild conditions. Bivariate signals can be understood and analyzed with concepts from optics (Poincaré sphere, Stokes parameters, etc.). In addition, their geometric phase is one of the "Pancharatnam-Berry" type. It is anticipated that multivariate signals should also possess a geometric phase under some conditions.

Project summary The proposed work aims at bridging the gap between the study of multidimensional quantum systems and signal processing theory. It will consist in studying and exploring the geometric phase of multivariate signals (i.e. vector-valued time series). The candidate will study existence conditions and will design algorithms for its practical estimation. Concepts from differential geometry (fiber bundles, connections, curvature, parallel transport, etc.) will be used to provide a model for the geometric phase description of multivariate signals. Simulations will be conducted to illustrate the cases where multivariate time series possess a geometric phase and potential applications in signal processing tasks (denoising, classification) will be investigated.

Candidate profile He/she should be enrolled in a M2R or engineer diploma in one or more of the following fields: theoretical physics, applied mathematics, signal and image processing. The candidate should have good writing and oral communication skills.

Environment Position can be started anytime from February, 2025 and duration is up to 6 months. The candidate will be based in Gipsa-Lab, Grenoble. This internship will be hosted by the RICOCHET ANR project, with regular meetings and exchanges with researchers from the project.

References

[1] M.V. Berry, "Quantal phase factors accompanying adiabatic changes", Proc. R. Soc. A 392, 45-57, 1984.

[2] A. Bohm, *et al.*, "The geometric phase in quantum systems", Springer, 2003.

[3] M.V. Berry, "The adiabatic phase and Pancharatnam's phase for polarized light", Journal of Modern Optics, Vol. 34, No. 11, 1987.

[4] N. Le Bihan, J. Flamant, and P.-O. Amblard, "The geometric phase of bivariate signals",EUSIPCO Conference, Lyon, 2024.